Study Packet for
Quantitative Reasoning Assessment
Hollins University

This packet includes:

- A set of 26 review problems
- Short solutions to the review problems
- Complete solutions to the review problems with links to an appropriate topic on Purplemath.com. Please note that this additional information may be helpful but it is not inclusive.

The problems are reflective of the general topics covered on the QR Assessment.

The solutions provided show one way to work these problems. In most cases there are many other valid ways to approach the problem.

If you do not know how to work some of the problems in this packet, you can review the provided solutions as well as any notes or books from high school.

1. Officials estimate that 320,000 Boston-area party-goers attended the 1995 Independence Day celebration on the banks of the Charles River. They also estimate that the party-goers left behind 40 tons of garbage. Given that a ton equals 2,000 pounds, how many pounds of garbage did the average party-goer leave behind?

2. Subway tokens cost 85 cents. How many can you buy with $20?

3. A student's grade depends on her score on four different exams. Her average on the first three exams is 92. What must she score on her fourth exam in order to guarantee a final average of at least a 90?

4. In 1990, a person places $1,000 in an investment that earned 10% interest, compounded annually. Calculate the value of the investment for the years 1991, 1992, and 1993.

5. According to *The New York Times*, scientists studying the atmosphere have recently detected a decrease in the level of methyl chloroform, a man-made industrial solvent that is harmful to the ozone layer. In 1990, the level of methyl chloroform was 150 parts per trillion (150 ppt), but by 1994 the level had fallen to 120 ppt. By what percentage did the level of methyl chloroform decrease between 1990 and 1994?

6. In 1990, according to census data, one in four Americans over eighteen years of age had never married, as compared to one in six in 1970. What is the ratio of the fraction of never married Americans in 1990 to the fraction of never married Americans in 1970?

7. One year ago, a person invested $6,000 in a certain stock. Today, the value of the investment has risen to $7,200. If, instead, the person had invested $15,000 one year ago instead of $6,000, what would the investment's value be today? (Assume that the $15,000 investment would increase by the same proportion as the $6,000 investment.)

8. Evaluate the following expressions given that $v = -2$ and $w = 3$.
   (a) $3\,(v - 2w)$ (b) $v^2 + w^2$

9. Figure 1 gives weight charts for baby boys and girls age birth to 18 months. Each chart gives weights for three different sizes of baby: 5^{th} percentile, or small babies, 50^{th} percentile, or average babies, and 95^{th} percentile, or large babies. For example, according to the chart, the 50^{th} percentile weight for a 6-month old girl is about 15 pounds, while the 95^{th} percentile weight for a 6-month old girl is about 18 pounds.

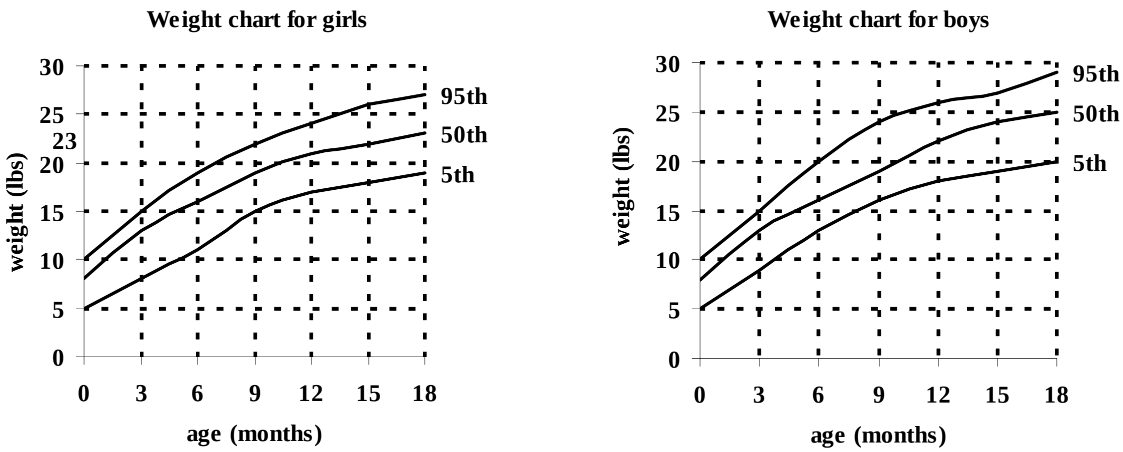

Figure 1: *Weight charts for baby boys and girls, age birth to 18 months*

(a) About how much more does an average eighteen-month-old boy weigh than an average eighteen-month-old girl?

(b) Consider two 6-month-old boys, one in the 5^{th} percentile and one in the 95^{th} percentile. About how old will the smaller boy be when he weighs as much as the larger boy does now? (You should assume that the smaller boy remains in the 5^{th} percentile as he grows.)

10. In 1993, there were 80 turtles living in a wetland. That year, the population began to grow by 12 turtles/year. Find a formula for P, the number of turtles, in terms of t, the number of years since 1993.

11. Find (a) the perimeter and (b) the area of the shape in Figure 2.

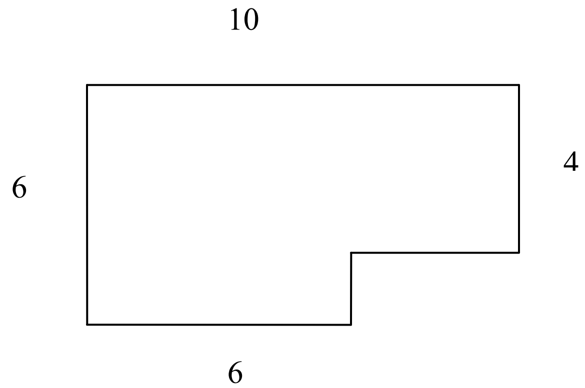

Figure 2

12. Figure 3 shows the number of movies with weekend receipts in different dollar ranges for the holiday weekend of June 30 - July 4, 1995. For example, according to the chart, two movies earned at least \$15 million but less than \$20 million. (They happened to be by *Judge Dredd*, at \$16.7 million, and *Mighty Morphin Power Rangers: The Movie*, at \$17 million.) According to Figure 3, how many movies earned more than \$10 million?

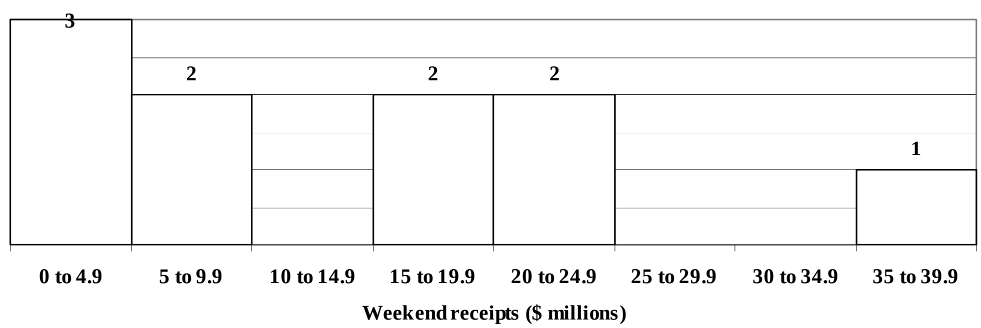

Figure 3: *The number of movies having total receipts in various ranges for the weekend of June 30 – July 4, 1995.*

13. Suppose you need to rent a car for one day and that you compare the cost at two different agencies. The cost (in \$) at agency *A* is given by $C_A = 30 + 0.22n$, where *n* is the number of miles you drive. Similarly, the cost at agency *B* is given by
$C_B = 12 + 0.40n$.

(a) If you drive only a few miles, which agency costs less, *A* or *B*?

(b) How far would you need to drive in order for the other agency to become less expensive?

14. According to the Cable News Network (CNN), the number of injured in-line skaters (or "roller-bladers") was 184% larger in 1994 than it was in 1993. Did the number of injured skaters almost double, almost triple, or more than triple?

15. For a certain flight out of Chicago, let P_f be the price of a first-class seat and P_c be the price of a coach-class seat. Furthermore, let N_f be the number of first-class seats and N_c be the number of coach-class seats available on the flight. Assuming that every available seat is sold, write an expression in terms of these constants that gives the value of *R*, the total amount of revenue (money) generated by the airline for this flight.

16. Solve the equation $\frac{Z_1}{Z_2} = \frac{K_1}{K_2}$ for K_2.

17. There are 0.6 grams of powder in a dish. One-fifth of the powder spills out of the dish. How many grams of powder are left in the dish?

18. Find the value of the two pointers shown in Figure 4.

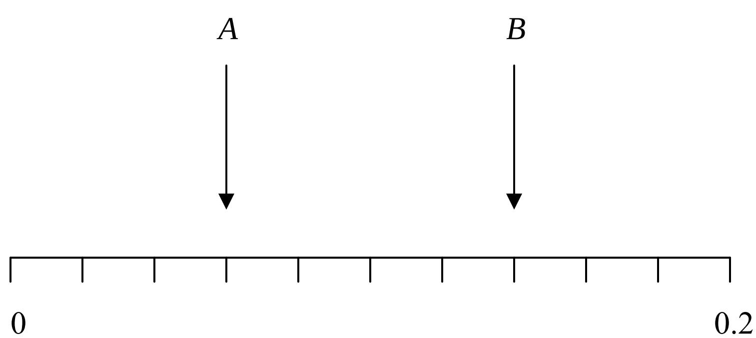

Figure 4

19. A six-foot tall man is walking home. His shadow on the ground is eight feet (ft) long. At the same time, a tree next to him casts a shadow that is 28 feet long. How tall is the tree?

20. Graph the equation $5s + 15 - t = 0$ on the set of axes provided in Figure 5. Label the s- and t-intercepts.

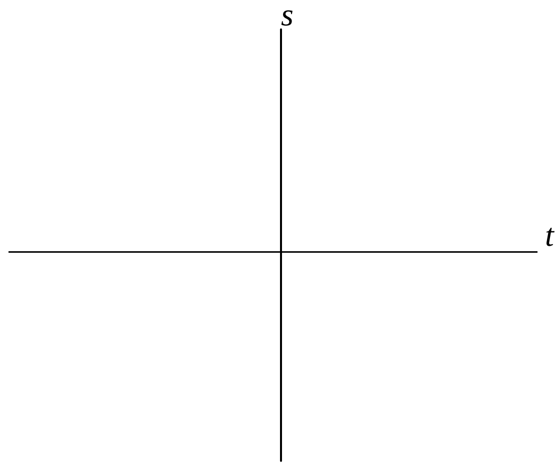

Figure 5

21. Evaluate the following. Express your answers in scientific notation.

(a) $(2 \times 10^{-4})(3 \times 10^{5})$

(b) $\dfrac{(3 \times 10^{-4})(8 \times 10^{5})}{4 \times 10^{-7}}$

22. The equations below describe several different animal populations over a period of time. In the equations, P stands for the size of the population and t stands for the year. Match the appropriate equation or equations to the verbal descriptions that follow.

(i) $P = 1000 - 50t$

(ii) $P = 8000\,(0.95)^t$

(iii) $P = 1000 + 70t$

(iv) $P = 5000 + 2000 \sin(2\pi t)$

(a) This population decreases by 5% each year.

(b) This population increases by the same number of animals each year.

(c) This population rises and falls over the course of the year.

(d) In year t = 0, these two populations are at the same level.

23. Match the following equations to the graphs shown in Figure 6.

(a) $y = 6 - 0.8x$ (b) $y = 6 + 0.9x$

(c) $y = 4 + 0.5x$ (d) $y = 8 + 0.5x$

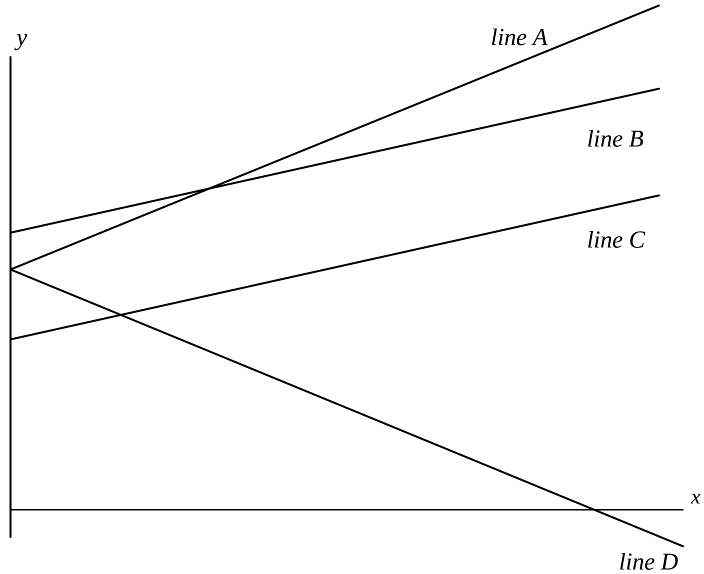

Figure 6

24. Figure 7 gives the rate (in thousands of gallons per minute) that water is entering or leaving a reservoir over a certain period of time. A positive rate indicates that water is entering the reservoir and a negative rate indicates that water is leaving the reservoir. State all time intervals on which the *volume* of water in the reservoir is increasing.

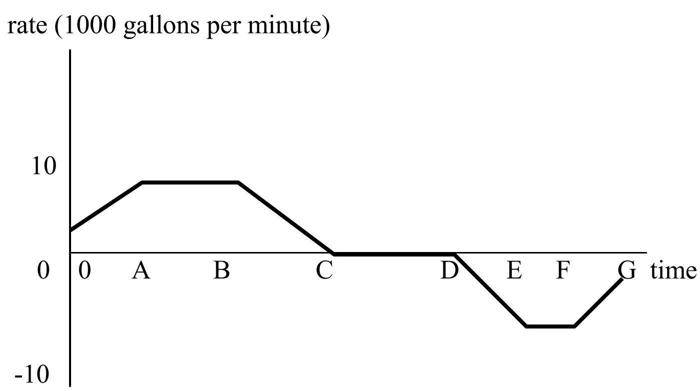

Figure 7

25. For the 2003 season, 24,000 people visited the Blue Lagoon Water Park. The park was open 120 days for water activities. Attendance was greatest on July 4th when 500 people visited the Park, and attendance was lowest on the closing day when only 50 people came to the Park. Determine the following values or explain why it is not possible to do so from the given information:

(a) The mean number of visitor per day

(b) The median number of visitors per day

(c) The mode number of visitors

26. Decide whether the following statement makes sense or does not make sense. Explain your reasoning.

"I have seen about 10^{50} commercials on TV."

!

Short Answers to QR Study Packet

1. $\frac{1}{4}$ pound
2. 23 tokens
3. 84
4. \$1100 in 1991, \$1210 in 1992, and \$1331 in 1993
5. 20%
6. 3/2 or 1.5
7. \$18,000
8. (a) –24 (b) 13
9. (a) About 2 lbs more (b) About 18 months old
10. $P = 80 + 12t$
11. (a) perimeter is 32 (b) area is 52
12. 5 movies
13. (a) Agency *B* (b) more than 100 miles
14. almost triple
15. $R = N_f P_f + N_c P_c$
16. $K_2 = K_1 Z_2 / Z_1$
17. 0.48 grams
18. Pointer *A* reads 0.06 and pointer *B* reads 0.14.
19. 21 feet tall
20. The *s*-intercept is –3 and the *t*-intercept is 15. See figure 8 on page 20.
21. (a) 6.0×10^1 (b) 6.0×10^8
22. (a) ii (b) iii (c) iv (d) i and iii
23. (a) line *D* (b) line *A* (c) line *C* (d) line *B*
24. From time 0 to time *C*
25. (a) 200 visitors a day

    (b) There is not enough information. The median would be the number of visitors on the average of the 60 & 61st day arrange in order from lowest attendance to highest.

    (c) There is not enough information to calculate the most frequent attendance.
26. Does not make sense

Solutions to QR Study Packet

1. The party-goers left behind 40 tons of garbage, each ton weighing 2000 pounds (lbs). That makes

$$40 \text{ tons} \times \frac{2{,}000 \text{ lbs}}{\text{ton}} = 80{,}000 \text{ lbs.}$$

   We can find the average amount of garbage that each person left behind by dividing the 80,000 pounds of garbage among the 320,000 party-goers:

$$\frac{80{,}000 \text{ lbs}}{320{,}000 \text{ people}} = \frac{1}{4} \text{ pound/ person}$$

   Thus, the average party-goer left behind ¼ lb of garbage.

   Problem solving: http://www.purplemath.com/modules/translat.htm
   Unit conversion: http://www.purplemath.com/modules/units.htm

2. Since $20 ÷ 0.85 is 23.5 (to one decimal of accuracy), this means you can buy 23 tokens with $20 and expect some change. Another way to work this problem is to think of buying tokens in sets of 10. One set of 10 tokens costs 10 × $0.85 = $8.50. This means that two sets of 10 tokens would cost 2 × $8.50 = $17. So if you buy 20 tokens for $17, you still have $3 left. This is only enough to buy 3 more tokens, because 3 × $0.85 = $2.55. In conclusion, $20 will buy 23 tokens and leave you with some change ($0.45, to be exact).

   Problem solving: http://www.purplemath.com/modules/translat.htm

3. Think about how you calculate an exam average: The point total of all of your exams is divided by the number of exams. Since this student's average for her first 3 exams is 92 points, this means that her point total for these 3 exams is 3 × 92 = 276 points. If you're not sure about this step, notice that

$$\begin{aligned}\text{Exam average} &= \frac{\text{total score}}{\text{no. exams}} \\ &= \frac{276}{3} \\ &= 92,\end{aligned}$$

   which is what we wanted. Now, if the student wants her final average for all 4 exams to be at least 90 points, then her point total for all 4 exams must be at least 4 × 90 = 360. Since she already has 276 points, she only needs 360 – 276 = 84 points more. Thus, she must score at least 84 points on her last exam.

   Finding the average (mean):
   http://www.purplemath.com/modules/meanmode.htm

4. Table 1 shows the bank balance for 1990, 1991, 1992, and 1993.

Year	Balance
1990	\$1000
1991	\$1100
1992	\$1210
1993	\$1331

Figure 1: *Bank balance over time (see Question 4).*

To find the balance for 1991, we begin with the 1990 balance of \$1000 and then add 10%:

$$\begin{aligned}\text{Balance in 1991} &= \text{balance in 1990} + 10\% \text{ of balance in 1990}\\ &= \$1000 + 10\% \text{ of } \$1000\\ &= \$1000 + 0.10 \times \$1000 \qquad \text{because } 10\% = 0.10\\ &= \$1000 + \$100\\ &= \$1100.\end{aligned}$$

Similarly, to find the balance for 1992, we begin with the 1991 balance of \$1100 and then add 10%. Notice that 10% of \$1100 is not the same as 10% of \$1000, and so this time the balance goes up by a different amount:

$$\begin{aligned}\text{Balance in 1992} &= \text{balance in 1991} + 10\% \text{ of balance in 1991}\\ &= \$1100 + 10\% \text{ of } \$1100\\ &= \$1100 + 0.10 \times \$1100\\ &= \$1100 + \$110\\ &= \$1210.\end{aligned}$$

Finally, to find the balance for 1993, we begin with the 1992 balance of \$1210 and then add 10%:

$$\begin{aligned}\text{Balance in 1993} &= \text{balance in 1992} + 10\% \text{ of balance in 1992}\\ &= \$1210 + 10\% \text{ of } \$1210\\ &= \$1210 + 0.10 \times \$1210\\ &= \$1210 + \$121\\ &= \$1331.\end{aligned}$$

Compound interest: http://www.purplemath.com/modules/expofcns4.htm

5. The formula for percent change -a *very* useful formula to know- is

$$\textit{Percent change} = \frac{\textit{amount of change}}{\textit{original amount}}$$

Here, the level of methyl chloroform changes by 30 ppt (parts per trillion) dropping from its original level of 150 ppt to its current level of 120 ppt. Using our formula, we see that

$$\begin{aligned} \text{Percent decrease} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{30 \text{ ppt}}{150 \text{ ppt}} \\ &= 0.20 \\ &= 20\% \qquad \text{converting from a decimal to a percent} \end{aligned}$$

Note that in general, to convert a decimal to a percentage, we shift the decimal point two places to the right. (Likewise, to convert a percentage to a decimal, we shift the decimal point two places to the left.) Here, we converted the fraction 30/150 to the decimal 0.20 by dividing:

$$\begin{aligned} \frac{30}{150} &= \frac{1}{5} \qquad \text{reducing the fraction} \\ &= 1 \div 5 \qquad \text{dividing} \\ &= 0.20 \end{aligned}$$

We then converted the decimal 0.20 to the percentage 20% by moving the decimal point two places to the right.

Converting decimals and fractions to percents:
http://www.purplemath.com/modules/percents.htm
Percent Change: http://www.purplemath.com/modules/percntof3.htm

6. The fraction of never married Americans in 1990 is $\frac{1}{4}$, and the fraction of never married Americans in 1970 is $\frac{1}{6}$. The ratio of the first fraction to the second is given by

$$\begin{aligned} \text{ratio} &= \frac{1/4}{1/6} \\ &= \frac{1}{4} \times \frac{6}{1} \qquad \text{multiply by reciprocal} \\ &= 6/4 \\ &= 3/2 \text{ or } 1.5 \end{aligned}$$

Recall that to divide by a fraction like $\frac{1}{6}$, we must multiply by the fraction's reciprocal, which means that we flip the fraction "upside down" and then multiply.

Ratio: http://www.purplemath.com/modules/ratio2.htm

7. Thinking in terms of proportionality, we see that the ratio of the stock's value today to its value one year ago ($15,000) should equal the rate ratio of $7,200 to $6,000

$$\frac{\text{value today}}{\$15{,}000} = \frac{\$7{,}200}{\$6{,}000}$$

$$\begin{aligned}
\text{value today} &= \frac{\$7{,}200}{\$6{,}000} \times \$15{,}000 && \text{multiplying} \\
&= \frac{6}{5} \times \$15{,}000 && \text{reducing fraction} \\
&= 6 \times \$3{,}000 && \text{dividing by 5} \\
&= \$18{,}000
\end{aligned}$$

Thus, the investment would be worth $18,000.
Another way to solve this problem is to think in terms of percentages. Using our formula for percent change (see Question 5), we have

$$\begin{aligned}
\text{Percent change} &= \frac{\text{amount of change}}{\text{Original amount}} \\
&= \frac{\$7{,}200 - \$6{,}000}{\$6{,}000} \\
&= \frac{\$1{,}200}{\$6{,}000} \\
&= 0.20 \\
&= 20\%
\end{aligned}$$

Thus, if the person had invested $15,000, it would grow by 20%:

$$\begin{aligned}
\text{Value of investment now} &= \$15{,}000 + 20\% \times \$15{,}000 \\
&= \$15{,}000 + 0.20 \times \$15{,}000 && \text{because } 20\% = 0.20 \\
&= \$15{,}000 + \$3{,}000 \\
&= \$18{,}000,
\end{aligned}$$

which is the same answer that we got before.

Solving proportions: http://www.purplemath.com/modules/ratio4.htm

8.

(a) We have

$$\begin{aligned}3\,(v-2w) &= 3\,((-2)-2\,(3))\\ &= 3\,(-2-6)\\ &= 3\,(-8)\\ &= -24\end{aligned}$$

(b) We have

$$\begin{aligned}v^2+w^2 &= (-2)^2+(3)^2\\ &= (-2)(-2)+(3)(3)\\ &= 4+9\\ &= 13.\end{aligned}$$

Evaluating an expression: http://www.purplemath.com/modules/evaluate.htm

9.

(a) From Figure 1, we see that the average 18-month-old girl weighs 23 pounds (lbs), and that average 18-month-old boy weighs 25 lbs. Thus, the boy was 2 lbs more than the girl.

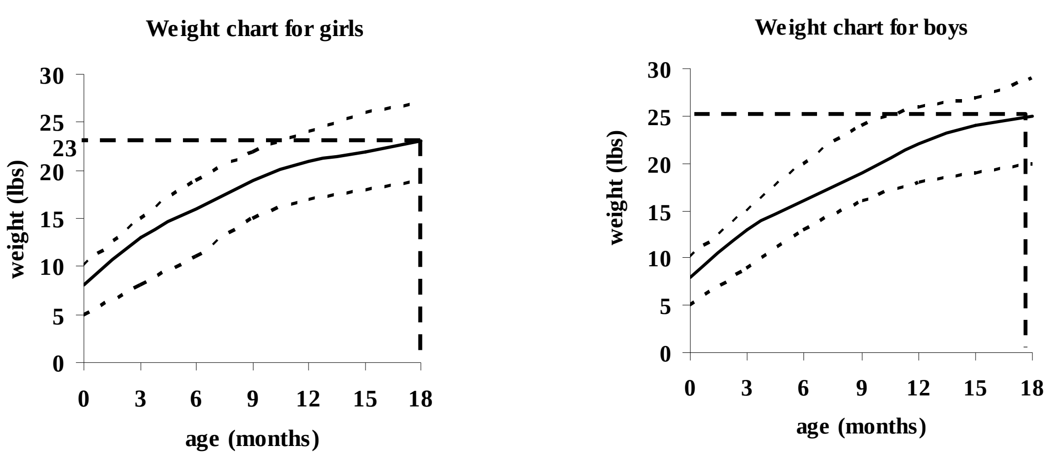

Figure 1: *For part (a) of Question 9, we see that the average 18-month-old girl weighs 23 lbs and that the average 18-month-old boy weighs 25 lbs.*

(b) From Figure 2, we see that the large boy weighs 21 lbs at 6 months of age, and that the small boy won't weight this much until he is 18 months of age.

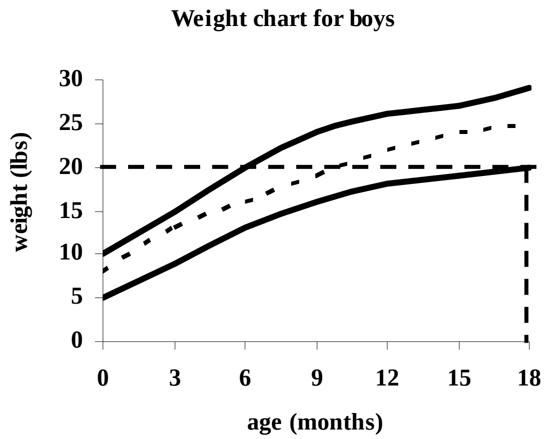

Figure 2: *For part (b) of Question 9, we see that a 5th percentile, 18-month-old boy weighs the same as a 95th percentile, 6-month-old boy.*

:

10. We have

$$\underset{\text{after } t \text{ years}}{\text{No. turtles}} = \underset{\text{in 1993}}{\text{no. turtles}} + \underset{\text{since 1993}}{\text{additional turtles}}$$
$$= 80 + \underbrace{12 + 12 + \ldots + 12}_{\text{12 more turtles each year}}$$
$$= 80 + 12 \times \text{no. years since 1993}$$
$$= 80 + 12\,t$$

Thus, a formula for P is $P = 80 + 12t$.

Another way to work this problem is to notice that the number of turtles is growing at a constant rate over time, which means that the equation for P will be linear, so that $P = b + mt$ where b and m are constants. Here, b is the initial value, or 80, and m is the growth rate, or 12. This gives us $P = 80 + 12t$, the same answer that we got before.

Using a Linear model: http://www.purplemath.com/modules/slopyint.htm

11.

(a) The perimeter of a shape is the distance around its border. The given shape has two unlabeled sides. From Figure 3, we see that these sides measure 2 and 4. Thus, by adding up all the sides, we see that the perimeter of the shape is given by

$$\text{Perimeter} = 6 + 10 + 4 + 4 + 2 + 6 = 32.$$

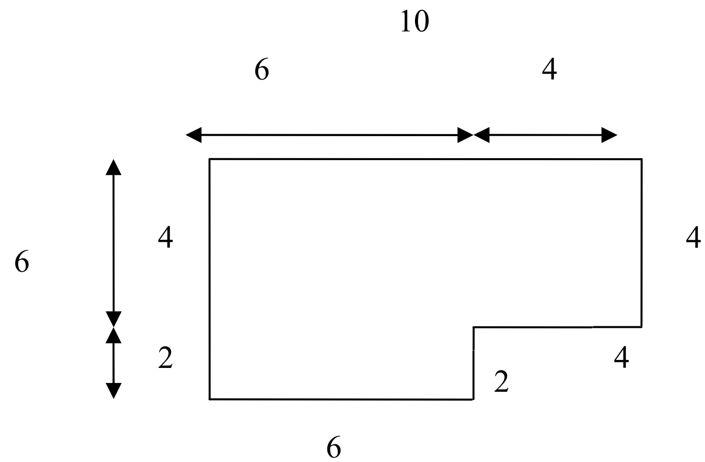

Figure 3: *To find the perimeter for part (a) of Question 11, first find the length of each edge.*

(b) From figure 4, we see that the shape can be broken into two different squares, one of side 6 and one of side 4. The area of a square is given by

$$\text{Area of square} = (\text{side})^2 = \text{side} \times \text{side}.$$

This means that the area of the square of side 6 is $6 \times 6 = 36$, and the area of the square of side 4 is $4 \times 4 = 16$, and so

$$\text{Area of shape} = 36 + 16 = 52.$$

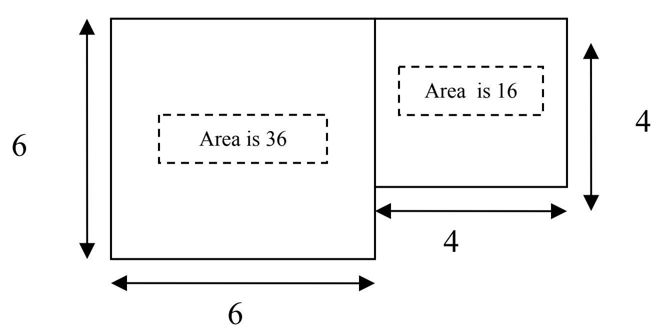

Figure 4: *To find the area for part (b) of Question 11, first break the shape into two squares.*

Finding area and perimeter: http://www.purplemath.com/modules/perimetr.htm

12. From Figure 5, we see that 5 of the top 10 movies made more than \$10 million on the weekend of June 30-July 4.

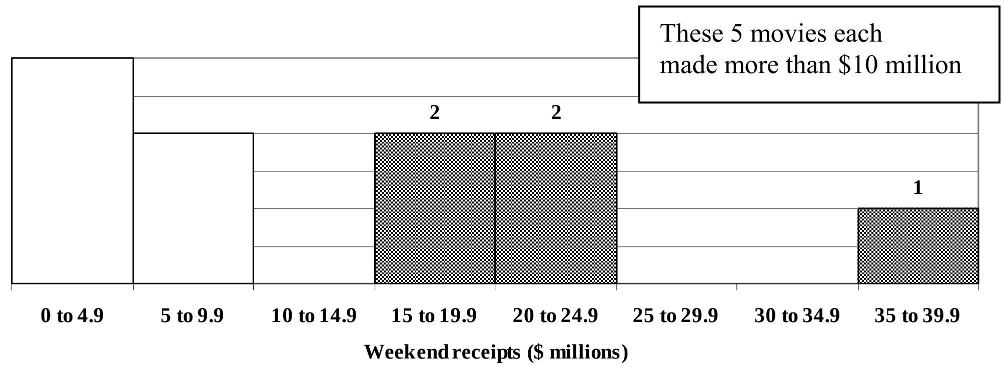

Figure 5: *For Question 12, we see that 5 of the top 10 movies (darkly shaded) made more than \$10 million.*

13.

(a) One way to work this problem is to plug in different distances for n to see which agency is less expensive. For example, suppose we imagine driving only 5 miles. In this case, the value of n would be 5, and we would have

$C_A = 30 + 0.22 \times 5 = 31.10$
$C_B = 12 + 0.40 \times 5 = 14.00$

Thus, the cost for driving 5 miles is \$31.10 at agency *A* but only \$14.00 at agency *B*. We conclude that agency *B* would cost less if we are going to drive only a few miles.
Notice that on the other hand if we imagine driving 200 miles, the costs work out differently:

$C_A = 30 + 0.22 \times 200 = 74$
$C_B = 12 + 0.40 \times 100 = 92.$

We see that to drive 200 miles would cost \$74 at agency *A* and \$92 at agency *B*. This means that to drive a long distance, it will cost less to rent from *A* than from *B*.

(b) At what distance should we switch agencies? In other words, at what distance does agency *A* cost no more than agency *B*? We can answer this by solving the equation $C_A = C_B$. Setting the formulas for these two costs equal to each other gives:

$$12 + 0.40n = 30 + 0.22n$$
$$0.40n - 0.22n = 30 - 12$$
$$0.18n = 18$$
$$n = 18/0.18 = 100$$

Thus, agency A will cost the same as agency B at $n = 100$ miles. This means that if we drive farther than 100 miles, we should rent from agency A instead of agency B.

Solving with linear equations: http://www.purplemath.com/modules/solvelin2.htm

14. If a quantity increases by 100%, it doubles in size. If it goes up by 200%, it triples in size; if it goes up by 300%, it quadruples in size; and so on. So, since the number of injured skaters increase by 184%, this means that the number of injured skaters more than doubled- and almost tripled- in size.

Meaning of percent: http://www.purplemath.com/modules/percntof.htm

15. We have

Total amount of money = amount for 1st class + amount for coach.

Now, suppose (just for the sake of argument) that 20 first-class tickets are sold for \$1000 each. This would mean that

Amount of money for first class = \$1000 per seat × 20 seats = \$20,000.

We see that to calculate the money brought in, we multiply the cost per seat by the number of seats. Since we aren't told the number of first-class seats or how much they cost, we must use the symbols N_f and P_f instead of 20 and \$1000, but the reasoning is the same:

Amount of money for 1st class = \$ P_f per seat × N_f seats = $N_f P_f$.

Similarly, the amount of money for coach is given by

Amount of money for coach = \$$P_c$ per seat ×N_c seats = $N_c P_c$.

Adding these two amounts together gives a formula for R, the total amount of money brought in by the airline for this flight:

R = amount for 1st class + amount for coach
= $N_f P_f + N_c P_c$.

Writing an expression: http://www.purplemath.com/modules/solvelit.htm

16. One way to work this problem is to flip or invert both sides of the equation and then multiply:

$$\frac{K_1}{K_2} = \frac{Z_1}{Z_2} \qquad \text{original equation (written backwards)}$$

$$\frac{K_2}{K_1} = \frac{Z_2}{Z_1} \qquad \text{flip both sides}$$

$$K_2 = K_1 \frac{Z_2}{Z_1} \quad \text{or} \quad \frac{K_1 Z_2}{Z_1} \qquad \text{multiply by } K_1$$

Another approach is more direct. First, because we are solving for K_2, we will multiply both sides by K_2 in order to clear it from the denominator:

$$\frac{Z_1}{Z_2} = \frac{K_1}{K_2} \qquad \text{original equation}$$

$$K_2 \frac{Z_1}{Z_2} = K_1 \qquad \text{multiply by } K_2$$

Next, divide both sides by the fraction Z_1/Z_2 in order to isolate K_2:

$$K_2 = \frac{K_1}{Z_1/Z_2}.$$

To simplify this result, we recall that to divide by a fraction, we must multiply by its reciprocal. Since the reciprocal of Z_1/Z_2 is Z_2/Z_1, we have:

$$K_2 = K_1 \frac{Z_2}{Z_1}.$$

This is the same answer that we got before.

Solving for a variable: http://www.purplemath.com/modules/solvelit.htm

17. Since one-fifth of the 0.6 grams is given by

$$\frac{1}{5} \times 0.6 = \frac{0.6}{5} = 0.12$$

we see that 0.12 grams of powder spill from the dish. Thus, there are 0.60 – 0.12 = 0.48 grams left in the dish. Alternatively, since one-fifth of the 0.6 grams spills out, four-fifths of the 0.6 grams remain. This means that there are

$$\frac{4}{5} \times 0.6 = \frac{2.4}{5} = 0.48 \text{ grams left.}$$

Multiplying fractions: http://www.purplemath.com/modules/fraction3.htm

18. The scale is divided into two large pieces. Together, the two pieces measure 0.20, and so each unit alone must measure 0.10. Each large piece is subdivided into 5 smaller pieces. This means that each small piece measures $0.10 \div 5 = 0.02$. Thus, the small tick marks on the scale are 0.02 units apart. (See Figure 6.) Pointer *A* is at the third tick mark which means it reads $3 \times 0.02 = 0.06$. Pointer *B* is at the seventh tick mark which means that it reads $7 \times 0.02 = 0.14$.

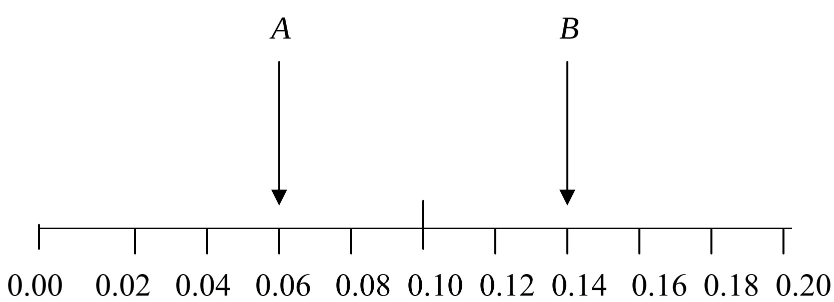

Figure 6: *The tick marks on the scale from Question 18 are 0.02 units apart.*

Use of fractions: http://www.purplemath.com/modules/fraction.htm

19. From Figure 7, we see that the man, the tree, and the shadow form two similar triangles. In the figure, *x* stands for the height of the tree, which is what we would like to determine. Since the triangles are similar, the ratios of corresponding sides are equal. In other words, the ratio of the tree's height to it shadow's length, *x*/28, equals the ratio of the man's height to his shadow's length, 6/8. This gives us the equation *x*/28 = 6/8, which we can solve for *x*:

$$\begin{aligned} \frac{x}{28} &= \frac{6}{8} && \text{by similar triangles} \\ x &= \frac{6}{8} \times 28 && \text{solving for } x \\ &= \frac{3}{4} \times 28 && \text{reducing} \\ &= 21. \end{aligned}$$

This means that the tree is 21 feet tall.

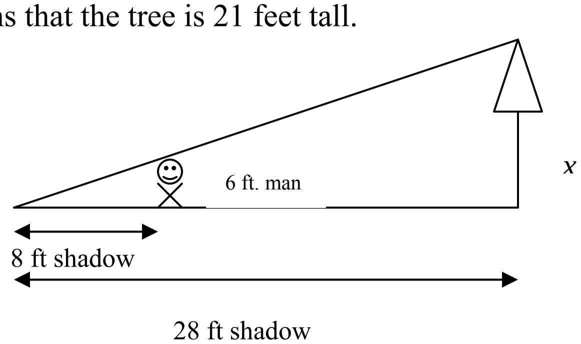

Figure 7: *The man and tree from Question 19 form two similar triangles.*

Similar triangles and proportionality: http://www.purplemath.com/modules/ratio6.htm

20. If we recognize that this equation is linear, then we know that its graph will be a straight line. We can find the s-intercept by setting $t = 0$, which gives:

$$\begin{aligned} 5s + 15 - 0 &= 0 \qquad \text{setting } t = 0 \\ 5s &= -15 \\ s &= -3. \end{aligned}$$

Similarly, we can find the t-intercept by setting $s = 0$, which gives:

$$\begin{aligned} 5 \times 0 + 15 - t &= 0 \qquad \text{setting } s = 0 \\ 15 - t &= 0 \\ t &= 15. \end{aligned}$$

Thus, the s-intercept is the point (0, –3) and the t-intercept is the point (15,0). Plotting these two points, we can draw a line passing through them to find the graph of the equation. (See Figure 8.)

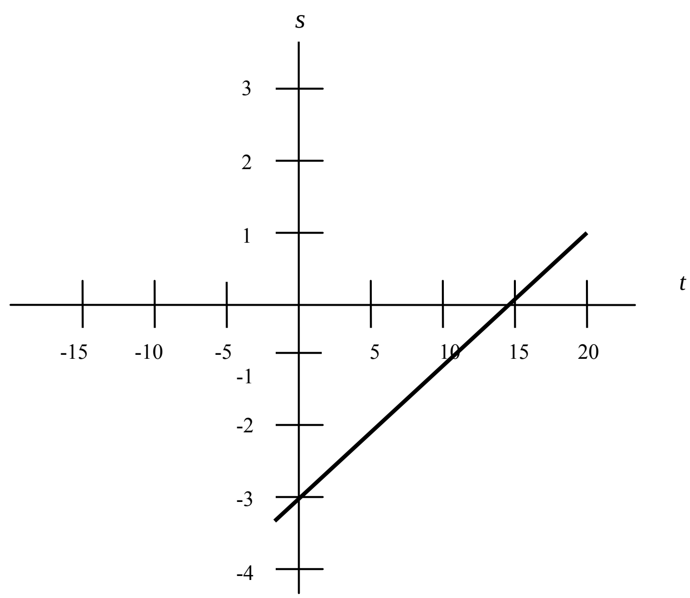

Figure 8: *The graph of the equation from Question 20 is a straight line.*

Another approach is to first place this equation into slope-intercept form– in other words, to write it as $s = mt + b$ where m is the slope and b is the s-intercept (the vertical intercept). Solving for s gives

$$\begin{aligned} 5s + 15 - t &= 0 \\ 5s + 15 &= t \\ 5s &= t - 15 \\ s &= 1/5\,(t - 15) \\ &= 1/5t - 3. \end{aligned}$$

Thus (as we have already seen) the s-intercept is $b = -3$. The slope is $m = 1/5$. To find the t-intercept, we set $s = 0$ and solve. We obtain $t = 15$ as before.

Graph of a line: http://www.purplemath.com/modules/slopgrph2.htm

21.

(a) We have

$$\begin{aligned}(2 \times 10^{-4})(3 \times 10^{5}) &= 2 \times 3 \times 10^{-4} \times 10^{5} \\ &= 6 \times 10^{-4+5} \\ &= 6 \times 10^{1},\end{aligned}$$

which is the same as 60.

(b) We have

$$\begin{aligned}\frac{(3 \times 10^{-4})(8 \times 10^{5})}{4 \times 10^{-7}} &= \frac{3 \times 8 \times 10^{-4} \times 10^{5}}{4 \times 10^{-7}} \\ &= \frac{24}{4} \times \frac{10^{-4+5}}{10^{-7}} \\ &= 6 \times \frac{10^{1}}{10^{-7}} \\ &= 6 \times 10^{1-(-7)} \\ &= 6 \times 10^{8},\end{aligned}$$

which equals 600,000,000 or 600 million.

Using Scientific notation: http://www.purplemath.com/modules/exponent3.htm

22. Equations (i) and (iii) are both linear. The slope of (i) is –50, and so this population decreases (goes down) by 50 animals per year. The slope of (iii) is +70, and so this population increases by 70 animals per year. Both populations start out at the same level, 1000. We can see this by setting *t* equal to 0 in these equations.

On the other hand, equations (ii) and (iv) are not linear. Equation (ii) is exponential, and equation (iv) is sinusoidal. If we try several different values for *t* in equation (ii), such as $t = 0$, 1, 2, and 3, we see that this population goes down by 5% each year:

$$\begin{aligned}&8000\,(0.95)^{0} = 8000 \\ &8000\,(0.95)^{1} = 7600 \qquad \text{a 5\% decrease} \\ &8000\,(0.95)^{2} = 7220 \qquad \text{a 5\% decrease} \\ &8000\,(0.95)^{3} = 6859 \qquad \text{a 5\% decrease}\end{aligned}$$

By process of elimination, we see that statement (d) must go with equation (iv). This makes sense, because sinusoidal quantities rise and fall over time.

Putting all of this together, we see that statement (a) goes with equation (ii), statement (b) goes with equation (iii), statement (c) goes with equation (iv), and statement (d) goes with equations (i) and (iii).

Linear equation: http://www.purplemath.com/modules/slope.htm

23. The key to this problem is understanding what the values of m and b tell us about the graph of the linear equation $y = mx + b$. A positive value of the slope m corresponds to a line that rises when read from left to right, while a negative value of m corresponds to a line that falls. The larger the value of m, either positive or negative, the steeper the line. The value of the y-intercept, b, determines where the line crosses the y-axis.

Since equations (a) and (b) have the same y-intercept ($b = 6$), they must cross the y-axis at the same point. Moreover, equation (a) has a negative slope (–0.8) while equation (b) has a positive slope (+0.6). Notice that lines *A* and *D* have the same y-intercept and that line *A* climbs while line *D* falls. Thus line *A* corresponds to equation (b) while line *D* corresponds to equation (a).

On the other hand, equations (c) and (d) have the same slope (+0.5), and thus they describe lines of the same steepness, which is to say they describe parallel lines. Moreover, the y-intercept of equation (c) is less than 6, while the y-intercept of equation (d) is more than 6. This is significant because lines *A* and *D* cross the y-axis at 6. Thus, we know that equation (c) describes a line that crosses the y-axis at 6. Thus, we know that equation (c) describes a line that crosses the y-axis below lines *A* and *D*. Similarly, equation (d) describes a parallel line that crosses the y-axis above lines *A* and *D*. Thus, line *C* corresponds to equation (c) while line *B* corresponds to equation (d).

Linear equation: http://www.purplemath.com/modules/slope.htm or
http://www.purplemath.com/modules/slopgrph.htm

24. This question is tricky because it is easy to mistake the given graph for a graph of the reservoir's volume. But it is not a graph of the volume; it is a graph of the *rate* that water is entering or leaving the reservoir. We are told that water is entering the reservoir when the rate is positive, which means that the volume is increasing when the rate is positive. Thus, the answer is time 0 to time *C*, because on this interval (and at no other time) the rate is positive.

25.

(a) Another word for *mean* is average. The formula for an average is:

$$\frac{\text{sum of the numbers}}{\text{\# of numbers}}$$

So, for this question the *mean* or average is:

$$\frac{24{,}000 \text{ people}}{120 \text{ days}} = 200 \text{ people per day}$$

(b) The median number of visitors would be the "middle" value of the sample. In other words, in order to calculate the median, we would need to have the number of visitors for every day, then we would have to order them from least to greatest. Next, we would need to find the value in the middle. This is impossible to do from the given information, because we do not have the number of visitors for all 120 days the park was open.

(c) The mode is that value that occurs most frequently in our sample. In order to find the mode, we would have to have the number of visitors for every day the park was open. Since we don't have this information, we cannot find the most frequently occurring number.

Mean, median & mode: http://www.purplemath.com/modules/meanmode.htm

26. This statement does not make sense. A human lifetime is only a few billion (10^9) seconds. (Do the unit conversion to see that 1 billion seconds is about 30 years.) Thus, to see even a billion commercials you'd have to see one almost every second of your entire life. Remember that 10^{50} is much larger than 10^9

Meaning of exponents: http://www.purplemath.com/modules/exponent.htm